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On the Structure and Motions of Jupiter's Red Spot

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A new hypothesis—called the Cartesian diver hypothesis—is proposed to explain the physical nature and observed variations in longitude, size and intensity of Jupiter's Great Red Spot. In the light of recent laboratory studies of phase behavior in light gas mixtures, it is suggested that the Red Spot is a region of contrast in the cloud structure of Jupiter's outer layers, caused by the presence of a mass of hydrogen-rich solid floating within a fluid layer of hydrogen and helium at some depth below the visible surface. It is shown that this floating solid would exhibit many of the characteristics of a Cartesian diver. Equations of motion for a Cartesian diver in a rotating system are derived, which suggest that the longitudinal motion of the Red Spot consists of several oscillatory components of different amplitudes and frequencies. These predictions are in agreement with motions determined from recent precise measurements of Red Spot longitude. Mechanisms for the source of the observed oscillations are discussed, and estimates are made of the extent of the underlying vertical motions. Effects of the rapid rotation of the planet upon the dynamic behavior of the fluid in the vicinity of the Cartesian diver are qualitatively included, and the resulting physical model is used to explain not only the observed variations of size and intensity of the Red Spot, but also the manner in which these variations correlate with changes in longitude. Laboratory studies and further observations designed to assist in verifying the Cartesian diver hypothesis are suggested.

INTRODUCTION

Systematic surveillance of Jupiter's Red Spot has been carried out since about 1878, although reliable records of its observation extend at least as far back as 1831. A survey of 17th-century documents relating to Jupiter has led Chapman (1968) to conclude that the Red Spot was first observed by Giovanni Cassini around 1666, and that Cassini, rather than Robert Hooke, should be credited with its discovery. This mysterious object is thus known to have existed for at least 140 years, very probably for at least 300 and perhaps very much longer.

In spite of the attention that it has received, the Red Spot remains almost as much of an enigma today as when it was first observed and sketched by Cassini three centuries ago. Most of the speculation about it has centered on attempts to answer the question: what is it? Among other questions that one may ask are: why is it unique? why is it (sometimes) red? why does it appear to drift in longitude while remaining nearly fixed in latitude? what accounts for changes in its size and appearance? how can one explain its observed motions?

The dearth of clues to the understanding of the Red Spot is reflected in the paucity of proposed explanations which have been advanced over the years. Of the proposed explanations aimed at identifying the Red Spot the two which have received most attention are: (1) that it is a "raft" of solid or fluid material floating on or within an ocean consisting of a liquid or dense gas; and (2) that it is the top of a more or less stagnant column of fluid, called a Taylor Column, which forms within a fluid flowing past an obstacle in a rotating system.

In view of the observed motions of the Red Spot the idea that it is a detached object floating in a fluid atmosphere is an attractive one. The evidence for a slow and apparently random drift in longitude has been summarized by Peek (1958), on the basis of observations extending over a period of 120 years. The drift was evident much earlier, however, and according to Peek the floating raft hypothesis was first advanced by G. W. Hough in 1881, although there was no evidence at that time concerning the composition of the raft or the ocean in which it floated. Peek suggested that the raft consists of two different phases of the same solid, and that it floats, not on a liquid surface, but within a fluid layer at a level at which its average density is equal to that of the surrounding fluid. Mounting evidence that Jupiter is composed largely of hydrogen and helium undermined the idea of a solid raft, however, since there was no known solid light enough to float in an atmosphere composed of the two lightest elements. Peek (1958) reviewed all the evidence for the raft hypothesis and concluded that the most promising material for the solid was helium, which might exist in two or more solid phases at high pressures. However, DeMarcus (1958) has pointed out that at high pressures the density of helium exceeds that of hydrogen by a factor of three or more, so that solid helium is almost certainly more dense than any fluid mixture of hydrogen and helium.

DeMarcus and Wildt (1966) proposed the novel alternative of a floating raft which is fluid rather than solid. Citing recent experimental evidence for fluid–fluid phase separations in multicomponent mixtures of supercritical gases, they suggested that the Red Spot is a region where phase separation occurs under the condition that the two distinct phases have the same density but different compositions. While fluid–fluid phase separations are known to occur in binary mixtures such as helium–ammonia (Tsiklis, 1952) and helium-methane (Streett and Hill, 1970), there is as yet no evidence that coexisting fluid phases of equal density occur in a multicomponent mixture of these gases. The existence of a fluid raft would seem to require somewhat larger concentrations of heavier gases, such as ammonia and methane, than are presently suggested for Jupiter's atmosphere.

The Taylor Column hypothesis of the Red Spot was proposed by Hide (1961) on the basis of estimates of certain parameters required to characterize the dynamics of a fluid layer of a rapidly rotating planet. He concluded that a "topographical feature" on the solid planet, having a height of only a few kilometers, can produce a Taylor Column, and that the visible Red Spot is the top of such a column. In subsequent papers (Hide, 1963, 1966*a*, 1966*b*, 1968; Hide and Ibbetson, 1966; Hide, Ibbetson, and Lighthill, 1968) the theory was critically examined from many points of view, and laboratory and theoretical studies on Taylor Columns were reported. Hide has been cautious in assessing the results of these efforts, and has constantly stressed the need for additional information on the Jovian interior, especially its hydrodynamic and magnetic properties. In response to criticisms that the topographic features would have to migrate to accommodate the observed motion of the Red Spot, Hide has been careful to point out that the "topographical feature" which produces the Taylor Column is not necessarily a mountain or crater on a solid surface, but that it may be a "hydromagnetic disturbance" (Hide, 1966*b*) or an "electric current loop" (Hide, 1966*a*). He has assumed that the observed drift in the longitude of the Red Spot reflects changes in the rotation rate of the planet, and has suggested the reversible transformation of rotational energy into magnetic energy as a possible cause. It is evident from the foregoing that the observed motions have generated a corresponding mobility in the Taylor Column hypothesis. It has been necessary to introduce plausible qualitative processes the implications of which are not contradictory to the observations. Nevertheless, in spite of some criticism (Stone and Baker, 1968; Goody, 1969) the Taylor Column hypothesis has remained the only explanation of the Red Spot developed beyond the level of simple conjecture.[1]

Since the observed motions represent real data which eventually must be explained by a theory of the Red Spot, we shall now turn to a discussion of these motions.

Observations of the Red Spot

The most comprehensive source of information on the visible features of Jupiter is the book by Peek (1958), which summarizes and interprets all of the observational data up to about 1947. It includes a considerable amount of speculation, of which that relating to the Red Spot anticipates many of the ideas presented here.[2]

[1] The paucity of explanations has not been a barrier to lively discussions of the Red Spot, and at least one other explanation deserves mention here. At a recent conference on planetary atmospheres, a particularly heated discussion of the merits of the Taylor Column hypothesis was in progress when R. Hide paused to describe a "new theory." Its author is a science reporter for an English newspaper, who, in attempting to find a simple descriptive analogy for the Taylor Column, has somehow seized upon the phrase "whirlpool of motion." Through a fortuitous typographical error his printed story reported that the Red Spot is, "a whirlpool of emotion in the atmosphere of Jupiter." This is known as the emotional whirlpool hypothesis.

[2] Bertrand M. Peek was a Cambridge-educated school teacher and a lifelong amateur astronomer —an amateur at least in the sense that astronomy was not the source of his livelihood. A good part of his life was apparently spent in observing Jupiter. The single-mindedness with which he pursued his hobby is revealed in a paper (Peek, 1941) in which, in a classic example of British understatement, he reported, "One observation, recorded on October 17 [1940], ... was made during an air raid under conditions of atrocious seeing and is clearly, from the record, of very little value."

Peek's chapter on the observational data for the Red Spot is devoted mainly to a discussion of its appearance and its interaction with other visible features in the same latitudes. The main point of interest here is his plot of the wanderings of the Red Spot longitude, which is reproduced in Fig. 1. Peek has suggested that the graph shows a form of periodic motion, with a period of about 50 years; however, it is important to note that in this plot the mean rotation period for the planet has been taken as that which reduces the net wanderings to a minimum, and that small changes in the assumed period would alter the shape of the curve. In any case, the tendency of the Red Spot to undergo a slow drift in longitude is clearly shown, and it is worth noting that, on this plot at least, the rates of eastward and westward drift are approximately the same.

A wealth of new information on the motions of the Red Spot, and on other surface features, has been obtained during the past decade by Reese, Solberg, and others at New Mexico State University (Solberg, 1968*a*, 1968*b*, 1969*a*, 1969*b*; Solberg and Reese 1969; Reese and Smith, 1968; Reese and Solberg, 1969; Reese, 1969*a*, 1969*b*). Since 1963 the Red Spot has been photographed on every opportunity at the New Mexico State University Observatory. With refined measurement techniques the position of the Red Spot has been measured to within about $\pm 0^\circ.2$ (Solberg, 1969*b*). Most of the data obtained in this program have been reported in this journal, and only the main conclusions concerning changes in longitude need be summarized here.

The photographic data, combined with earlier visual transit data, show that since 1937 the longitude of the Red Spot has increased at a fairly constant rate. This is essentially a continuation of the slow drift described by Peek. The more precise recent observations show, however, that the motion includes several additional components, of approximately periodic nature, superimposed on this slow drift (Solberg, 1968*a*). The most prominent of these are: (1) an oscillatory component with a period of approximately 9 years and

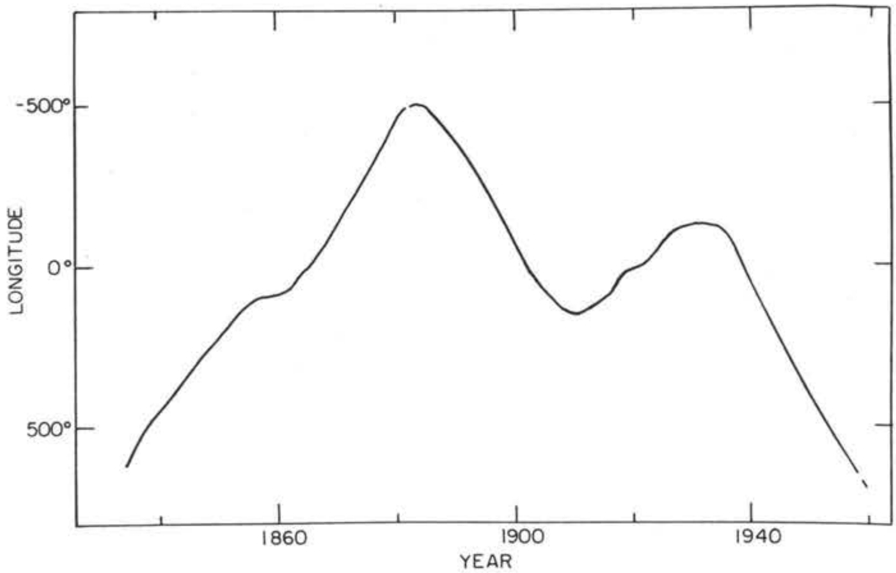

FIG. 1. Long term variations in the Red Spot longitude, after Peek (1958).

an amplitude of about 10°; (2) an almost sinusodial oscillation with a period of about 90 days and an amplitude of approximately 1°; and (3) suspected short-term oscillations having a period of the order of 1 day to 1 week and an amplitude less than 1°. In addition to these approximately regular motions, small discontinuities sometimes appear in plots of the longitude of the Red Spot vs time.

In Fig. 2 we have combined the plots of longitude vs time for the six apparitions from 1963 to 1969 (Solberg, 1968*a*, 1968*b*, 1969*a*, 1969*b*; Reese, 1969*a*, 1969*b*). The 9-year and 90-day period motions are clearly shown, together with somewhat irregular motions of shorter period. The latter motions are suspected to have a period of about one week.

A more comprehensive and suggestive display of observational data is given in Fig. 3 where we have plotted the longitude, latitude, length, and width of the Red Spot, as functions of time, for several apparitions (Solberg, 1968*a*, 1968*b*, 1969*a*; Reese, 1969*a*). Even a casual examination of these curves suggests some definite relation between changes in longitude, on the one

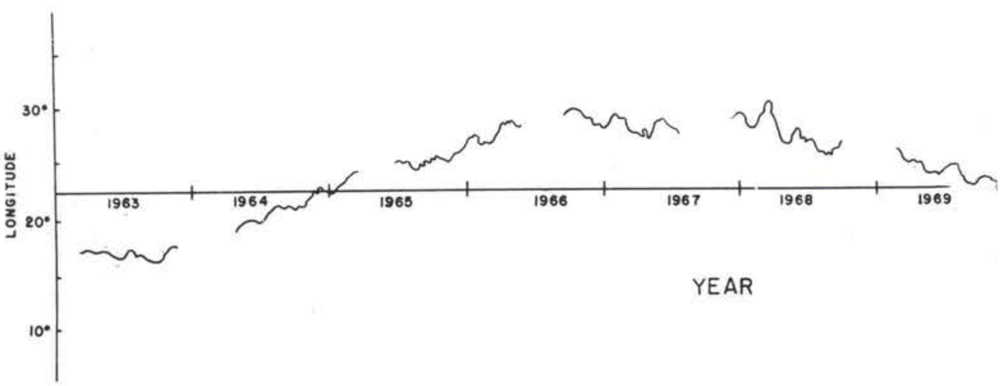

FIG. 2. Longitude of the Red Spot vs time for the six apparitions from 1963–69 (Solberg, 1968*a*, 1968*b*, 1969*b*; Reese, 1969*a*, 1969*b*).

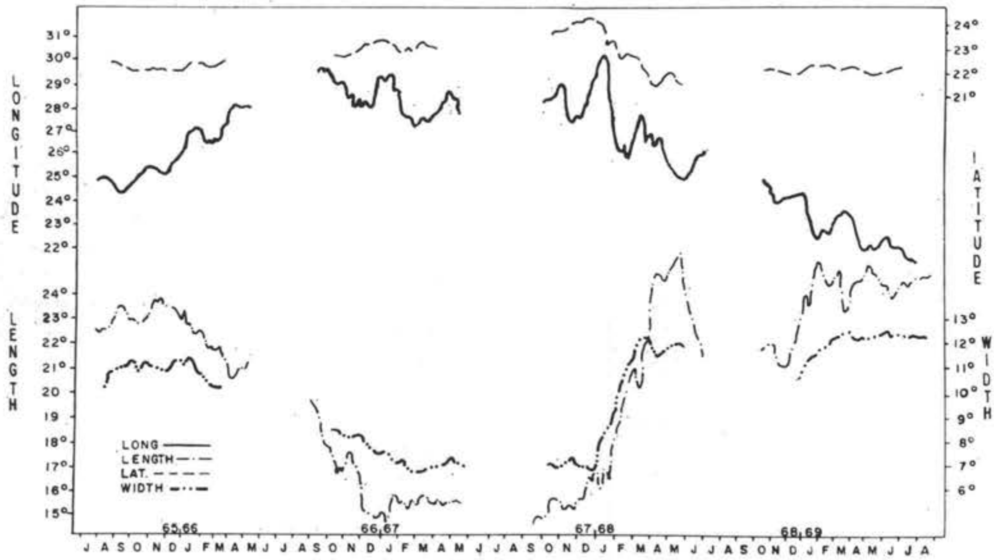

FIG. 3. Variations in longitude, latitude, length, and width of the Red Spot during four recent apparitions (Solberg, 1968*a*, 1968*b*, 1969*a*; Reese, 1969*a*).

hand, and changes in length and width on the other. In fact, there is strong evidence that the length curve includes oscillatory components of 9-year and 90-day periods which are 180° out of phase with the corresponding components of the longitude curve. In other words, as the longitude decreases, the length increases, and vice versa. There is a similar correlation between longitude and width, at least for the 9 year period. Short term changes in length and width are also present, but irregular spacing and scatter in the data effectively mask any regularity in this component.

Not quite so obvious, but equally interesting, is the fact that the slope of the length curve over the 9-year period is always about twice that of the width curve, indicating that the rate of change of Red Spot length is twice that of the width. We note that the same ratio holds for the magnitudes of the length and width (i.e., $L \simeq 2W$). The average length of the Red Spot is about 25,000 km.

We shall attempt to incorporate the foregoing features into a simple theoretical model of the Red Spot. In doing so we should point out that we are concentrating on explaining the observed motions by proposing a model which is consistent with the observations. We cannot suggest a real explanation for why the Red Spot is unique or why it exists within the latitude band that it does. Perhaps a topographic feature is present in that region to bring about a massive accumulation of solid hydrogen in the helium–hydrogen atmosphere so that the Cartesian diver (proposed in the following section as the cause of the Red Spot) is formed. Once formed such a feature could exist indefinitely. However, this is only a more or less plausible statement and not an explanation. We now turn to the basic hypothesis which we propose as a reasonable explanation for the existence of the Red Spot.

THE CARTESIAN DIVER MODEL

We present here a new hypothesis for the Red Spot which combines, at least in a qualitative way, some features of both a floating raft and a Taylor Column (though not, we trust, of an emotional whirlpool). It is called the Cartesian diver hypothesis, following DeMarcus and Wildt (1966) who used the term "thermodynamic Cartesian diver" to characterize their suggestion that the Red Spot is a floating raft of fluid.[3]

We propose that the Red Spot is a region of contrast in Jupiter's cloud structure, caused by a mass of hydrogen-rich molecular solid floating within a fluid layer of hydrogen and helium at some depth below the visible surface. This suggestion is based on laboratory studies of phase equilibria in light gas mixtures at high pressures, which provide strong evidence that a hydrogen-rich solid can exist deep in Jupiter's atmosphere, under the condition that it is neutrally buoyant at a fixed level in the stratified hydrogen–helium fluid. This floating solid would indeed behave as a Cartesian diver. It would seek an equilibrium level in the fluid, and this equilibrium level would change with changes in the physical and thermodynamic properties of the system (e.g., temperature, pressure, phase composition, etc.). Convective motions in the outer layers of Jupiter (Peebles, 1964; Hubbard, 1969) would be modified in the region of the floating solid, and these modifications would be reflected in the cloud structure at the visible surface.

Owing to the condition of thermodynamic equilibrium between solid and fluid phases, fluctuations in ambient conditions are likely to result in a kind of dynamic physical equilibrium as well, characterized by a continuous cycle of solidification and melting. According to this picture the floating solid is not necessarily continuous, but is more likely an assembly of solid chunks whose total mass and size are continually changing.

Estimates of temperature in the Jovian atmosphere, and of the melting curve of solid hydrogen at high temperatures and pressures, suggest that the solid exists at depths of several thousand kilometers. It is unlikely that this solid would be visible from the outside, but its influence on patterns of convection would almost certainly appear in the cloud structure at the visible surface. This is generally consistent with the suggestion of Ingersoll and Cuzzi (1969) that Jupiter's bright zones are regions of warm, rising fluid, while the dark belts are regions of cool, sinking fluid. Other dynamical and hydrodynamical considerations, not unlike those invoked by Hide in the Taylor Column hypothesis, can also account for the visibility of the Red Spot. Hide (1968) has in fact pointed out that a Taylor Column could form above a floating raft, but has not followed up this idea in seeking to explain the Red Spot motions.

The physical basis for the Cartesian diver hypothesis has been discussed in detail elsewhere (Streett, 1969) so that we shall confine ourselves to a short review of the processes that are involved.

Melting Behavior in Hydrogen–Helium Mixtures

Current knowledge of the melting behavior of pure substances and its relevance to planetary interiors has been reviewed by Wildt (1961). The available evidence suggests that the melting temperature rises indefinitely with increasing pressure, so it is possible, at least in theory, to solidify any molecular substance, at any temperature, by the application of sufficient pressure.[4] The maximum pressures and

[3] A Cartesian diver is a device, attributed to Descartes, which illustrates several laws and principles of physics. In its simplest form it consists of an inverted float or "diver," containing a small amount of trapped air, in a container of water. As the air bubble is compressed, the diver descends and it can be made neutrally buoyant at a fixed level within the water, given the proper balance between its weight and the volume of trapped air. For an ordinary mechanical Cartesian diver the neutrally buoyant level is an unstable equilibrium point. Changes in certain physical properties of the system (temperature, pressure, weight of diver, etc.) cause the diver to seek an new equilibrium level.

[4] In this discussion we restrict our attention to phase transitions which are likely to occur in the region of stability of the molecular phase. However, the possibility that the Red Spot is somehow connected with the pressure-induced metallic transition in hydrogen cannot be ruled out. Current estimates put this transition pressure at several million atmospheres (DeMarcus, 1958; Schneider, 1969). Pressures in

temperatures at which the melting curves of pure hydrogen and helium have been experimentally studied are: for hydrogen, 5600 atm and 80°K (Wooley, Scott, and Brickwedde, 1948); and for helium, 14,000 atm and 77°K (Langer, 1961). Over the known range, the melting pressure of helium, at a fixed temperature, is about twice that of hydrogen and extrapolations of the melting curves suggest that this general relation holds at higher temperatures.

Little is known of the melting behavior of systems of more than one component. Surveys of the melting behavior of solidified gases (Bridgman, 1958; Stewart, 1965) indicate that no significant study of the melting behavior of systems of two or more components has been published. Theoretical methods are also of little value here. In general the laws of thermodynamics and chemistry cannot predict, even qualitatively, the phase behavior of multicomponent systems over broad ranges of pressure and temperature. Experimental studies of the phase behavior of hydrogen–helium mixtures have been confined mainly to the liquid-vapor region at low temperatures ($T < 33°K$) and pressures below about 100 atm (Streett, Sonntag, and Van Wylen, 1964; Greene and Sonntag, 1968; Sneed, Sonntag, and Van Wylen, 1968). It is therefore necessary to resort to empirical evidence concerning the probable melting behavior of this system at high pressures.

The phase rule of Gibbs fixes the number of independent variables for a system having a set number of phases and components, and thereby provides a framework for the empirical classification of heterogeneous systems. In recent years, experiments on gas–liquid phase equilibria in binary mixtures of light gases have been carried out at pressures as high as 10,000 atm, and certain patterns in the phase behavior of these systems have begun to emerge. In two recent papers Streett (1969, 1970) has reviewed the available phase data for light gas mixtures and has suggested several possible extrapolations of the hydrogen–helium phase diagram to high pressures. There is strong evidence that the solidification of hydrogen–helium mixtures takes place in such a way that a reversal in the sign of the difference in density between coexisting fluid and solid phases occurs on crossing a fixed boundary in pressure–temperature space. A summary of this evidence is presented here.

Two assumed features of the hydrogen–helium phase diagram are of paramount importance to the Cartesian diver hypothesis. These are: (1) that the pressure induced transition from fluid to solid in a hydrogen–helium mixture occurs over a finite range of pressures at a fixed temperature; and (2) that in the region of transition a hydrogen-rich solid phase exists whose density can be less than, equal to, or greater than that of the coexisting fluid, depending on the difference in helium content between the two phases.

The first of these features follows from the most fundamental generalization of the phase rule as it applies to a two-phase, two-component system. Briefly stated, its implication for planetary structures is this: if a fluid–solid transition occurs at some depth below the visible clouds of the Jovian planets, then a region of transition exists in which solid and fluid phases can coexist in thermodynamic equilibrium. It follows that layered structures are likely to result from gravitational separation of these phases. This point has been neglected in the formulation of models for Jovian interiors. In cases where the possible existence of a solid surface has been discussed (DeMarcus, 1958; Öpik, 1962) the implicit assumption has been made that helium is either absent from the region of solidification or that its presence does not affect the solidification of hydrogen.

The second important feature of the hydrogen–helium phase diagram (concerning the relative densities of coexisting solid and fluid phase) derives not from a fundamental physical principle, but from an unusual property of the hydrogen–helium system. This system belongs to a class of binary mixtures in which the

this range are estimated to exist at depths of 15,000 to 20,000 km below the visible surface of Jupiter (DeMarcus, 1958).

more volatile component (helium in this case) has the higher molecular weight, and is therefore more dense, at high pressures, than the second component, even though the former may be a gas and the latter a liquid or solid in the pure state. The significance of this fact was first observed by Onnes (1906) when he compressed helium gas over liquid hydrogen at 20°K. At a pressure of about 30 atm he found that the helium rich gas phase became more dense than the liquid phase and sank to the bottom of the vessel.

The equality of density between two phases reduces the number of independent variables of the system to one, so that the inversion pressure is a unique function of temperature in the two phase region. Sneed *et al.* (1968) have experimentally determined the locus of this phase density inversion over a narrow range of pressures and temperatures in the gas–liquid region. Although the density inversion has not been experimentally observed in gas–solid mixtures of hydrogen and helium, there can be little doubt that it does occur. Similar density inversions have been observed in nitrogen–ammonia (Krichevskii, 1940), carbon dioxide–water (Takenouchi and Kennedy, 1964), neon–methane (Streett and Hill, 1970), and other mixtures.

The simplest means of illustrating the principles of interest here is a phase diagram. Since we are interested primarily in effects due to pressure and composition, the phase behavior is best described by isotherms drawn on pressure–composition diagrams. It is to be emphasized that diagrams representing phase behavior along a path for which temperature increases with pressure—which more closely resembles a vertical sounding of the Jovian atmosphere—would have the same general form. Hence the following discussion applies equally to isothermal atmospheres and to those heated from the bottom.

Figure 4 shows a proposed isotherm for the hydrogen–helium system for a temperature, T_1, above the critical temperature of hydrogen. Points a and b are the melting points of pure hydrogen and helium, while the areas S_1, S_2, and F are regions in which homogeneous single phases exist. These are a fluid phase F, a hydrogen-rich solid S_1, and a helium-rich solid S_2. The remaining areas, $S_1 + S_2$, $S_1 + F$, and $S_2 + F$, are two-phase regions. At a pressure P_2, for example, any mixture of overall composition between c and d separates into a solid phase of composition c and a fluid phase of composition d. At pressures above P_4 any mixture is entirely solid, but may consist of a single solid phase, S_1 or S_2, and/or a mixture of these two phases, depending on the overall composition.

The sequence of phase states likely to be encountered in descending into a hydrogen–helium atmosphere, of overall composition X_1, lie along the vertical line 1-2-3 in Fig. 4. At pressures below P_1 (point 2) the mixture consists of a homogeneous fluid phase F, while at pressures above P_4 (point 3) it is entirely solid. At intermediate pressures (P_2, P_3) the mixture separates into solid and fluid phases S_1 and F.

Figure 4 shows the form of a diagram for a binary system in which the components are completely miscible in the fluid phase and partially miscible in the solid phase. (There is some evidence that hydrogen and helium are only partially miscible in the fluid phase at high pressures, even at temperatures above the critical temperature of hydrogen (Sneed *et al.*, 1968; Streett, 1969). This would result in a more complex phase diagram, which might be reflected in discontinuous changes in composition and density within the fluid region of Jupiter.) The form shown here is based mainly on empirical evidence. The most important relevant data are those of Tsiklis (1946) for the solid–fluid phase behavior of binary mixtures of carbon dioxide with hydrogen, nitrogen, and methane at 273°K. The maximum pressure reached by Tsiklis was 7000 atm, which is roughly equivalent to P_3 in Fig. 4. He suggested that the complete diagrams would be similar to Fig. 4.

Recalling that a and b are the melting points of hydrogen and helium at T_1, let us now consider the relative densities of the co-existing phases represented by the

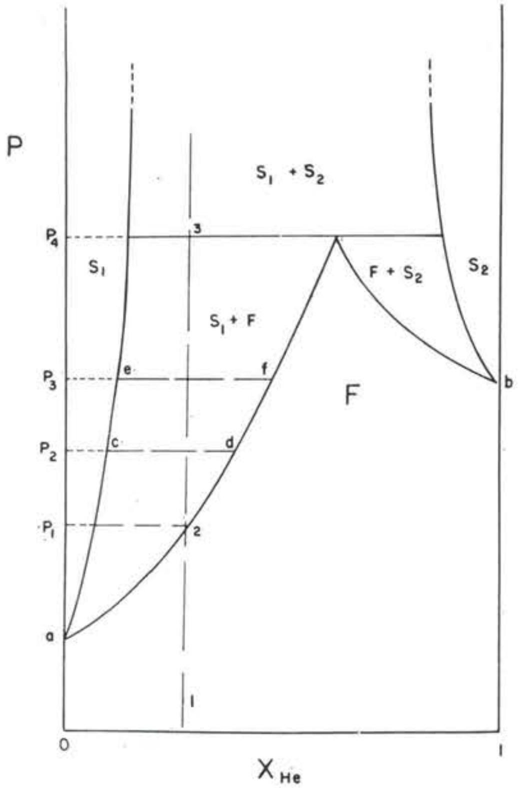

FIG. 4. Suggested pressure–composition diagram for hydrogen–helium, showing the regions of coexistence of solid and fluid phases at high pressures.

lines a-c-e and a-d-f. It is well known that all substances undergo an increase in density upon solidification at high pressures. Evidence for the persistence of significant density differences between fluid and solid phases along the melting curve, at high pressures, has been emphasized by Bridgman (1958). It is attributed to structural differences, on the molecular level, between a disordered fluid and a crystalline solid. Thus, in the two-phase region just above a (Fig. 4), the solid phase is likely to be more dense than the fluid phase because of the density increase accompanying solidification. As the pressure increases, however, the density of the fluid phase increases more rapidly because of its excess helium content. As a result there exists a pressure P_2 at which the two phases have equal densities. At lower pressures it is the solid, and at higher pressures the fluid, which has the greater density.

An estimate of the difference in helium concentration required to produce equality of density can be obtained from estimated hydrogen and helium densities at high pressures (DeMarcus, 1958), using the relation

$$\frac{1}{\rho_{\mathrm{m}}} = \frac{X_{\mathrm{H_2}}}{\rho_{\mathrm{H_2}}} + \frac{X_{\mathrm{He}}}{\rho_{\mathrm{He}}}, \qquad (1)$$

where ρ is the molar density, X is mole fraction, and subscript "m" refers to the mixture. Equation (1) states that the molar volumes of hydrogen and helium are additive. The data of DeMarcus indicate that the molar density of helium exceeds that of hydrogen by a factor of about 1.6 at high pressures. Substituting this relation into (1), along with the equation $X_{\mathrm{H_2}} + X_{\mathrm{He}} = 1$, leads to the following equation for the mixture density as a function of hydrogen concentration:

$$\rho_{\mathrm{m}} = 1.6\,\rho_{\mathrm{H_2}}/(0.6X_{\mathrm{H_2}} + 1). \qquad (2)$$

A graph of ρ_{m} vs composition is shown in Fig. 5, in which densities are normalized to $\rho_{\mathrm{H_2}} = 1$. DeMarcus also indicates that the density increase accompanying the solidification of hydrogen is about 7%. Figure 5 shows that a density change of

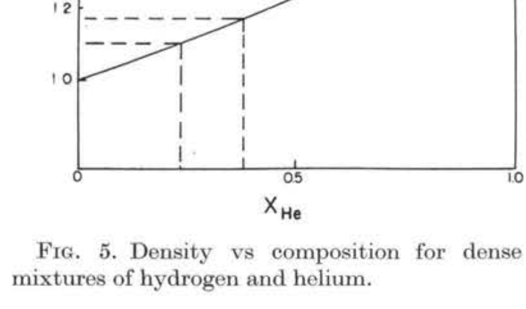

FIG. 5. Density vs composition for dense mixtures of hydrogen and helium.

this magnitude (the distance between the horizontal lines) is balanced by a difference in composition of about 15 mole % (the distance between the vertical lines).

Returning to Fig. 4, we note that if P_2 is the pressure at which the phases have equal densities, then at lower pressures it is the solid, and at higher pressures the fluid, which has the greater density. In a gravitational field the solid phase, which forms between the levels corresponding to P_1 and P_4, would gravitate toward the level corresponding to P_2, *both from above and from below*! Through slow diffusion its composition would approach the value c and it would remain suspended indefinitely in dynamic and thermodynamic equilibrium—a thermodynamically stabilized Cartesian diver. A continual cycle of melting and solidification might take place in a convective atmosphere, as a result of overturning in the region where the solid floats.

Because of uncertainties in the melting curves at high pressures and in the temperatures in Jupiter's outer layers, only crude estimates can be made of the depth at which solid hydrogen may be floating. In fact, if one extrapolates the hydrogen melting curve to high pressures by means of the Simon equation (DeMarcus, 1958) and adopts the temperature profile suggested by Hubbard (1969), the conclusion that solidification does not occur at all is strongly indicated. On the other hand, DeMarcus has shown that the assumption of an isothermal atmosphere ($T = 150°K$) provides an estimate of about 750 km for the minimum depth of solidification, while the assumption of an adiabatic (hydrogen) atmosphere suggests that solidification might occur at pressures of the order of one million atm and at temperatures of about 1000°K, corresponding to a depth of about 10,000 km. Smoluchowski (1970) has used a different approach to estimate that the solid surface lies at a depth of 4000 km below the visible surface. Thus we can only state that the Cartesian diver proposed here probably lies at a depth of several thousand km.

On the basis of this assumption we shall proceed to a discussion of plausible mechanisms which can generate oscillatory motions of the Red Spot.

Mechanisms for Oscillatory Motions

The argument that we shall use for analyzing the observed motions of the Red Spot is most simply stated as follows: Any physical process which causes the Cartesian diver to be displaced vertically from its equilibrium position at a given level will also involve a motion in longitude and/or latitude because of the rotation of the planet and the associated changes in the angular momentum of the diver. By treating the Cartesian diver as a particle subject to restoring and dissipative forces, we shall calculate the resulting motions.

In the following development the fluid dynamical pressure forces are assumed to be negligible. This assumption cannot be rigorously defended without a calculation of the circulation of Jupiter's atmosphere. However, qualitative justification is possible.

Ingersoll and Cuzzi (1969) have demonstrated that the large zonal currents between belts and zones of Jupiter are essentially geostrophic, implying the existence of large north–south pressure gradients. However, there are no observed north–south currents; hence from geostrophic balances effectively no east–west pressure gradients exist. From such considerations one cannot conclude that a sizable east–west pressure force acts on the diver. According to Reese and Smith (1968), the eastward flow along the southern edge of the Red Spot and the westward flow along the northern edge of the Red Spot are both 6100 km/day. Therefore, from Bernoulli's equation the net east–west pressure force that these currents exert on the Cartesian diver as a whole is zero.

Perhaps the strongest indication that pressure forces are not important is that the rigid particle analysis of the motion of the diver leads to results which are consistent with observation. The physical processes which are considered as important are listed below.

1. An obvious mechanism for restoring the Cartesian diver to its equilibrium level has already been mentioned in the previous section. Any vertical displacement of the diver will bring into operation the gravitational restoring force which arises from a difference in the rate of change of density with depth between the diver and the surrounding fluid. The buoyancy force (directed upward) is

$$-\int_V g(\rho_s - \rho_f)\,dV, \tag{3}$$

where V is the volume of the diver, subscripts "s" and "f" correspond to solid and fluid respectively, ρ is the density, g is the gravitational acceleration, and the integration is taken over the volume of the diver. Expanding ρ_s and ρ_f about the equilibrium level where the two densities are equal and keeping the lowest order contribution to the difference, $\rho_s - \rho_f$, yields

$$-\int_V g\left(\frac{\partial \rho_s}{\partial R} - \frac{\partial \rho_f}{\partial R}\right)(R - R_0)\,dV, \tag{4}$$

where R_0 is the radius at the equilibrium level. If we assume that the terms under the integral are essentially constant, (4) can be approximated by

$$-g(R - R_0)\left(\frac{\partial \rho_s}{\partial R} - \frac{\partial \rho_f}{\partial R}\right)V. \tag{5}$$

2. A second mechanism which can give rise to motions of the Red Spot (treated as a Cartesian diver) is given here as only a heuristic argument but its plausibility is based on the observation that the Red Spot, which lies in the South Tropical Zone, seems to be affected by the currents which bound it (the south component of the South Equatorial Belt flows westward on the northern side and the current between the south Temperate Belt and the South Tropical Zone flows eastward on the southern side). Solberg (1968*a*, 1969*b*) has pointed out that some transient spots in these currents circle the planet with a period of about 90 days relative to the Red Spot and has suggested a possible connection between the two. Reese and Smith (1968) in an analysis of observed vorticity in the Red Spot have estimated the velocities of spots in the two currents mentioned above and have also concluded that these spots circle the planet with a period of about 90 days.

If our basic assumption is correct, then it is reasonable to suppose that the boundary currents flowing past the northern and southern edges of the Cartesian diver will, via the pressure differences created by the Bernoulli effect, cause large pieces of the object to break off. Separation of large chunks from the Cartesian diver would affect its motion and, when the chunks become imbedded in these relatively fast currents, they provide visual evidence of the motion of the currents. Since the currents have a period of 90 days around Jupiter, a 90-day oscillation of the diver could be interpreted as an oscillation forced by the interaction of the diver with the currents on either side.

Furthermore, the south component of the South Equatorial Belt current accelerates as it flows past the northern edge of the Red Spot and the pressure difference due to the Bernoulli effect is consequently greater. Hence, there is a stronger likelihood that pieces would be broken off the northern edge of the Cartesian diver. The observation (Solberg, 1969*a*) that the northern edge of the Red Spot migrates in latitude considerably more than the southern edge supports this speculation.

3. A third mechanism which would affect the motion of the Red Spot is suggested by the fact that more heat is emitted than is incident on Jupiter by solar insolation. Hence, convection must be present. Ingersoll and Cuzzi (1969) have concluded that zones (light-colored bands) are regions of upward convection and belts (dark-colored bands) are regions of downward convection. The Red Spot is imbedded in the South Tropical Zone and the Cartesian diver assumption implies, therefore, that the region above the diver should shield the upper Jovian atmosphere from upward convection and the associated cloud structure. We suggest this as the reason for the darker color above the Red Spot.

Now consider the effect of convection on the Cartesian diver. Convection from the surface of the planet will carry heat upward to the base of the diver. In a nonrotating system the heat would be convected laterally to the edge of the diver and then be released to execute its natural vertical ascent. However, because of the rapid rotation of Jupiter the heat will be trapped under the object since a radial pressure gradient is balanced by an azimuthal flow in such a rapidly rotating system. Hence, the base of the diver will be heated and, as the heat accumulates, an increased buoyancy force is exerted on the diver. The diver will consequently rise to a higher level where conditions will be less appropriate for the solidification of hydrogen. These two processes will tend to erode the diver (presumably at its edges at first) and, as the erosion continues, the object will become mechanically less resistant to the stresses imposed by the buoyancy force. Hence, when the diver is in its highest position, it is susceptible to massive disintegration around its edges. When the latter occurs, much of the heat trapped at the base will be released and convected upward, an increased amount of cloudiness should be observed in these regions of upward convection, and the Red Spot should appear smaller and considerably less distinct. Also, as the disintegration occurs, chunks of smaller size ("spots" in the atmosphere) should be observed to move around Jupiter in the currents adjoining the Red Spot. After the release of heat the Cartesian diver will descend toward its equilibrium position and begin to grow by entraining more material again. The descent will continue, perhaps past the point of equilibrium, and the Spot should regain its former prominence and size as the clouds generated by the sudden release of heat begin to dissipate. The cycle is then repeated.

Although the foregoing argument is admittedly very speculative, the physical processes involved in the sequence of supposed events are reasonable. In any event, the behavior which has been described is consistent with the observations of the longer period (9-year) oscillations which the Red Spot executes, as we shall see later. We shall therefore assume that the Cartesian diver is subject to a force with a nine-year period because of the mechanism outlined. Without a very complicated analysis it is not possible to justify the choice of the nine-year period and we leave it here as an *ad hoc* assumption.

Equations of Motion of a Cartesian Diver

We consider first the motions of a Cartesian diver in a nonrotating system. Vertical motion of the diver in this system might result from: (1) the return of the diver to its equilibrium level following a temporary displacement; and (2) changes of the position of the diver brought about by dynamic or thermodynamic forces. In the first case the restoring force is given by Eq. (5) under the assumptions listed there and the system is analogous to a weight suspended from a spring in a viscous fluid. The resulting motion is a damped harmonic oscillation.

The second type of motion (forced changes in the level of the diver) might take any form; however, in the present context it seems reasonable to assume, for periods as short as those under consideration, that the average properties of Jupiter's outer layers have not changed significantly. Hence, any slow forced changes in the equilibrium level of the Red Spot might be expected to bring into play restoring forces which would generate motions of a roughly periodic nature.

If R represents the vertical position of a diver of mass m, with equilibrium position R_0, and if K is the ratio of the restoring force to the displacement, then the equation of motion for the natural oscillation of the system is

$$m(d^2 R/dt^2) + b(dR/dt) = -K(R - R_0), \tag{6}$$

where the term $b(dR/dt)$ represents a damping force proportional to the velocity of the solid body. The solution of (6) takes the form

$$R = R_0 + \epsilon e^{-\beta t} \cos \omega t \tag{7}$$

where ϵ and ω are the amplitude and frequency of the oscillation and β is the damping coefficient. The latter two parameters are defined by the equations

$$\beta = b/2m \tag{8}$$

$$\omega = (\pi\beta^2 + K/m)^{1/2}. \tag{9}$$

If the periodic changes in the equilibrium level (forced oscillations with amplitudes A_i and frequencies Ω_i) are added to the natural oscillations in Eq. (7), the result is

$$R = R_0 + \epsilon\, e^{-\beta t}\cos\omega t + A_1 \sin\Omega_1 t + A_2 \sin\Omega_2 t. \tag{10}$$

The rotation of the system is introduced by writing an expression for conservation of angular momentum. In this simple model we assume that the diver floats in the atmosphere of a rotating planet, but that it is constrained to remain in a fixed latitude. [It is well known (Peek, 1959; Solberg, 1968*a*, 1968*b*, 1969*a*) that the latitude of the Red Spot is nearly constant at about 22°, but that variations with a range of 2 or 3 degrees do occur. It may be that the Red Spot is constrained to a nearly fixed latitude by the strong east–west currents mentioned earlier.] In this case R is the distance of the diver from the center of the planet. The equation for the conservation of the component of angular momentum parallel to the axis of rotation reduces to

$$\frac{d}{dt}\left(mR^2\frac{d\phi}{dt}\right) = -b'R^2\left(\frac{d\phi}{dt} - \Omega_{\mathrm{J}}\right), \tag{11}$$

where ϕ is the angular position of the diver in the plane of its rotation, Ω_{J} is the rotational frequency of the planet, b' is the coefficient of resistance to horizontal motion (i.e., motion perpendicular to R), and R is defined by Eq. (10). The constant b' is analogous to b in Eq. (6), but the two are not necessarily equal since both the geometry of the diver and inertial effects due to the rotation of the system can cause differences in resistance to horizontal and vertical motion through the fluid. Then it is convenient to introduce the constant α, defined by $\alpha = b'/m$. The importance of α and β in determining the form of the solution is discussed below.

A complete solution to (11) is possible if the simplifying assumption is made that the vertical excursions of the diver, ϵ, A_1, and A_2, are small compared to R_0 which is approximately equal to the radius of the planet. In this case the harmonic terms in (10) are uncoupled and their lowest order effects on the solution to (11) can be considered separately. To facilitate comparison with the observed motions of the Red Spot, it is desirable to describe the horizontal motions of the diver with respect to a fixed meridian on the planet; that is, in terms of its longitude. The longitude is given by the expression $\phi - \Omega_{\mathrm{J}} t$, where Ω_{J} is the rotational frequency of the planet.

The term $\epsilon e^{-\beta t}\cos\omega t$ in Eq. (10) represents the natural oscillation of the diver in the vertical direction. The solution to (11) resulting from this term is:

$$\phi - \Omega_{\mathrm{J}} t = \frac{2\Omega_{\mathrm{J}}\epsilon}{R_0[(\beta-\alpha)^2 - \omega^2]} \times \{(\beta - \alpha)\cos\omega t - \omega\sin\omega t\}\, e^{-\beta t}. \tag{12}$$

It is important to recognize here that α and β are decay constants (units of t^{-1}), which describe the rate at which motions of the diver relative to the fluid are damped out by viscous and inertial forces. If either α or β is large compared to the natural frequency ω, then the system is overdamped and oscillations do not occur. If α and β are small compared to ω (the underdamped case), then natural oscillations do occur, and (12) reduces to

$$\phi - \Omega_{\mathrm{J}} t \simeq \frac{2\epsilon\Omega_{\mathrm{J}}}{R_0\,\omega} e^{-\beta t}\sin\omega t. \tag{13}$$

That is, the damped vertical oscillation produces a similar oscillation in longitude, and the two motions are 90° out of phase.

The solution to (11) resulting from the forced oscillations in (10) takes the following form:

$$\phi - \Omega_{\mathrm{J}} t = -\frac{2A_i\Omega_{\mathrm{J}}}{R_0(\Omega_i{}^2 + \alpha^2)^{1/2}} \times \sin(\Omega_i t + \gamma_i). \tag{14}$$

This equation shows that a forced vertical oscillation leads to an oscillation in longitude of the same frequency with a lag given by γ_i defined by

$$\tan \gamma_i = \frac{\Omega_i}{\alpha}. \quad (15)$$

The amplitude of the longitudinal oscillation is the coefficient of the sine term in (14), denoted here by θ_i. Then the ratio of amplitudes of longitudinal and vertical oscillations is given by

$$\frac{\theta_i}{A_i} = \frac{2\Omega_J}{R_0} \frac{1}{(\Omega_i^2 + \alpha^2)^{1/2}}. \quad (16)$$

Equations (15) and (16) show that the lag angle and the ratio of amplitudes are determined by the values of Ω_i and α. It is useful to consider here the two possibilities: $\Omega_i \gg \alpha$ and $\alpha \gg \Omega_i$. In the first case, $\gamma_i \simeq \pi/2$ and $(\Omega_i^2 + \alpha^2)^{1/2} \simeq \Omega_i$, so that (14) reduces to

$$\phi - \Omega_J t = -\frac{2A_i \Omega_J}{R_0 \Omega_i} \cos \Omega_i t. \quad (17)$$

The physical meaning in this case is that damping effects are small. Equation (17) indicates that the induced longitudinal oscillation leads the forcing function $(A_i \sin \Omega_i t)$ by 90°.

In the second case, $\gamma_i \simeq 0$ and $(\Omega_i^2 + \alpha^2)^{1/2} \simeq \alpha$, and (14) reduces to

$$\phi - \Omega_J t = -\frac{2A_i \Omega_J}{R_0 \alpha} \sin \Omega_i t. \quad (18)$$

Here damping effects are important, and the vertical and longitudinal motions are 180° out of phase.

For these two cases, the amplitude ratios defined by (16) are, respectively,

$$\left|\frac{\theta_i}{A_i}\right| = \frac{2\Omega_J}{R_0 \Omega_i} \quad (19)$$

and

$$\left|\frac{\theta_i}{A_i}\right| = \frac{2\Omega_J}{R_0 \alpha}. \quad (20)$$

Thus for motions of high frequency $(\Omega_i > \alpha)$ the ratio is determined by Ω_i, while for low frequency motions $(\alpha > \Omega_i)$ the ratio is determined by α. These relations are important for estimating the extent of the vertical displacements which underlie observed longitudinal oscillations of the Red Spot.

Finally, we can write a complete oscillatory solution to (10) and (11) in the form:

$$\begin{aligned} \phi - \Omega_J t = {} & \frac{2\epsilon \Omega_J}{R_0 \omega} e^{-\beta t} \sin \omega t \\ & - \frac{2A_1 \Omega_J}{R_0(\Omega_i^2 + \alpha^2)^{1/2}} \sin(\Omega_1 t - \gamma_1) \\ & - \frac{2A_2 \Omega_J}{R_0(\Omega_i^2 + \alpha^2)^{1/2}} \sin(\Omega_2 t - \gamma_2), \end{aligned} \quad (21)$$

where we have assumed $\omega^2 \gg \alpha^2 + \beta^2$. The first term in (21) represents a natural oscillation triggered by an initial displacement ϵ from the equilibrium level R_0. If the time interval between such displacements is small compared to the reciprocal of β, then these natural oscillations are essentially continuous. If it is further assumed that the frequencies are such that $\omega > \Omega_1 > \Omega_2$, then the motions described by the 3 terms on the right of (21) have the form shown in the lower three curves in Fig. 6. The sum of these terms (the upper curve in Fig. 6) then illustrates, qualitatively at least, the expected longitudinal motions of a Cartesian diver in a rotating system. Note the similarity between this curve and the longitude curves in Figs. 1–3.

Interpretation of Red Spot Motions in Terms of the Cartesian Diver Model

If the longitudinal motions of the Red Spot are those of a Cartesian diver, then the observed amplitudes and periods of these motions can be interpreted in terms of the physical parameters in Eq. (21). Although knowledge of the physical properties of the Jovian atmosphere is very meager, it is at least possible to obtain crude estimates of certain properties for the purpose of judging the overall suitability of the Cartesian diver model.

If the graph in Fig. 2 represents a combination of natural and forced oscillations of a floating object, it is of interest

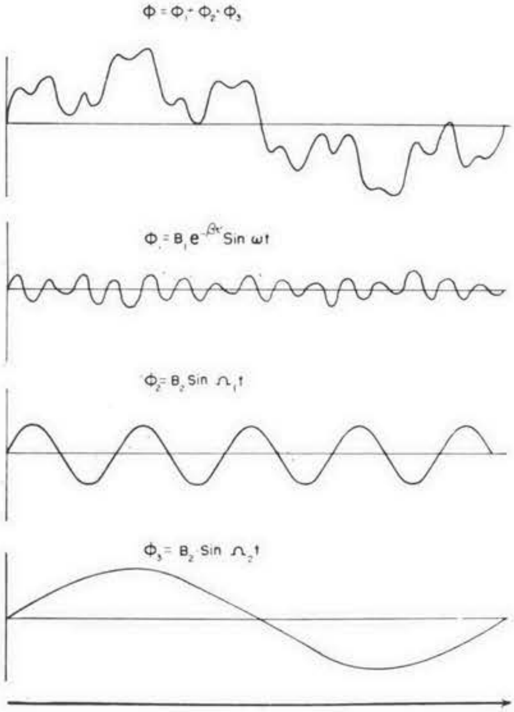

FIG. 6. The addition of several periodic functions of different amplitudes and frequencies results in a composite curve (upper curve) which bears a remarkable resemblance to the longitude–time curves for the Red Spot (Fig. 1–3).

to determine which of the components most likely represents the natural oscillation. According to Eq. (9) the natural frequency ω is approximately equal to $(K/m)^{1/2}$, where m is the mass of the solid object and K the ratio of the restoring force to displacement. For small displacements the restoring force is likely to result mainly from the difference in compressibility between solid and fluid phases. In this case the expression for K, taken from Eq. (5), is

$$K = gV\left[\frac{\partial \rho_f}{\partial R} - \frac{\partial \rho_s}{\partial R}\right]. \tag{22}$$

The density gradient terms are related to the compressibility C, given by

$$C = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right), \tag{23}$$

through the relation

$$\frac{\partial \rho}{\partial R} = \rho C \frac{\partial P}{\partial R}. \tag{24}$$

With values of 0.3 for ρ (DeMarcus, 1958) and 2500 cm/sec^2 for Jupiter's surface gravity, (24) reduces to

$$\frac{\partial \rho}{\partial R} \simeq 10^{-1} C \tag{25}$$

in cgs units. Bridgman (1958) reports a value for C of about 10^{-8} for hydrogen at a pressure of 13,000 atm. Further data of Bridgman (1958) on melting behavior at high pressures suggest that the difference in compressibility between solid and fluid phases is about two orders of magnitude smaller than the compressibility at these pressures ($\Delta C \simeq 10^{-10}$). Bridgman also emphasizes, however, that along the melting curve the difference in compressibility between solid and fluid phases decreases rapidly with increasing pressure. This is attributed to the fact that the compressibility of any substance, at high pressures, is effectively that of its molecules which have been compacted. Hence, at very high pressures the compressibilities of coexisting solid and fluid phases are nearly equal. Since the solidification of hydrogen in Jupiter's atmosphere is assumed to take place at pressures one or two orders of magnitude higher than the highest experimental pressures, it seems reasonable to assume a difference in compressibility about two orders of magnitude smaller than the value cited above. Applying this estimate to (22) gives

$$K = 10^{-13} gV. \tag{26}$$

Substituting (26) into (10), together with the relation $M = \rho V$, gives

$$\omega \simeq (K/M)^{1/2} = (10^{-13} g/\rho)^{1/2} \simeq 3 \times 10^{-5}\ \mathrm{sec}^{-1} \tag{27}$$

for the values $g = 2500$ and $\rho = 0.3$. The period P is

$$P = 2\pi/\omega = 2\pi/3 \times 10^{-5} \simeq 2 \times 10^5\ \mathrm{sec}, \tag{28}$$

which is about 2 days. It should be emphasized here that extrapolating the compressibility data to high pressures results in an uncertainty of at least an order of magnitude in either direction in the final estimate of P. We have also neglected the dynamic effects on fluid motions in a rotating system, which might exert a strong influence on the natural oscillations. Nevertheless, the results suggest that the apparent short period oscillation of the Red Spot is at least consistent with a natural oscillation, triggered perhaps by convective currents or other dynamic forces in the atmosphere.

Given this evidence, it is of interest to examine more closely the original observations of Red Spot longitude for signs of regularity in the short period motions. Using the original numerical data from the New Mexico State University Observatory (Reese, 1969*b*) we have, as a first step, tabulated the frequency of occurrence of different time intervals between apparent short term peaks in the longitude of the Red Spot. The resulting histogram (Fig. 7) suggests that components with periods of about 7 days and 12 days may be present. This is essentially the same conclusion reached by Solberg (1968*a*). Further analysis of the data, and probably more closely spaced observations at regular intervals, will be required before any conclusions can be reached concerning regularity in these motions.

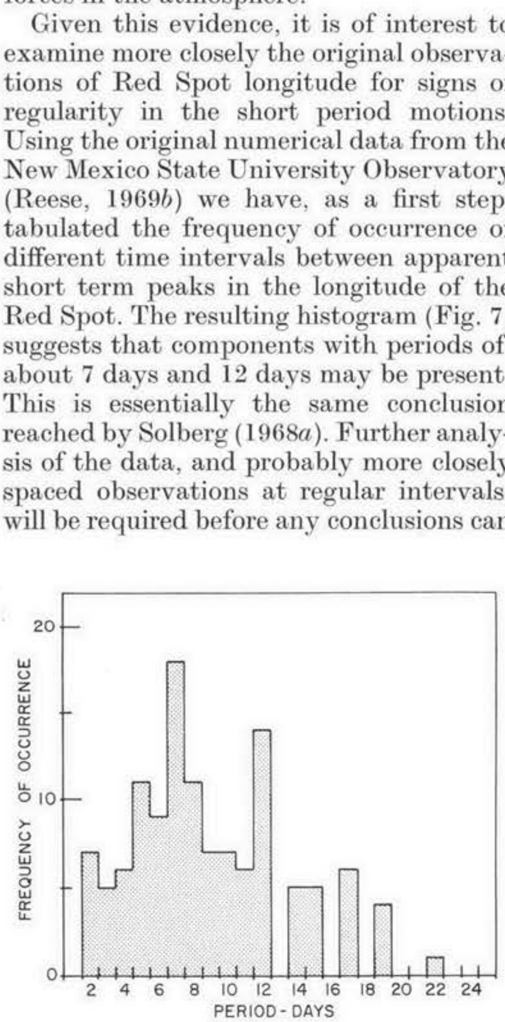

FIG. 7. Frequency of occurrence of various time intervals between apparent peaks in the short period oscillations of the Red Spot during 1962–69. These data suggest periods of 7 and 12 days.

The observed amplitude of the short period oscillation provides a means of estimating the extent of the corresponding vertical motion. We note that the coefficient of the first term on the right or Eq. (21) represents the amplitude of the longitudinal oscillation. Equating this expression to the observed value of about $0^\circ.3$, and assuming a period of 8 days, yields a value of about 10 km for ϵ. The total vertical excursion is twice ϵ, or about 20 km. The first derivative of (7) with respect to time gives an expression for the maximum vertical velocity $(dR/dt)_{\max} = \epsilon\omega \simeq$ 7.8 km/day. For longitudinal motion the amplitude is about 350 km and the maximum velocity is ~250 km/day (about 300 cm/sec). As these figures show, small vertical displacements can result in large horizontal motions as a result of Jupiter's rapid rotation.

The estimate $\epsilon \simeq 10$ km is based on the assumption that the decay constants, α and β, are small compared to ω, which, for an 8 day period, is about 0.8 rad/day. Since the thickness of the diver is likely to be small compared to its length and width, α is likely to be smaller than β. A crude estimate of β can be obtained if the force resisting the vertical motion of the solid is assumed to be the inertial force of the fluid flowing against the solid surface. In this case $b = \rho av/2$, where ρ is the density of the fluid, a is the area of the solid perpendicular to the direction of motion, and v is the velocity. The expression for β reduces to

$$\beta = b/2m = \rho av/4m = \rho av/4\rho ah = v/4h, \tag{29}$$

where h is the vertical thickness of the solid. If v is 8.5 km/day, any value of h greater than about 20 km results in a β which is at least an order of magnitude smaller than ω. Since the length and width of the Red Spot are of the order of 10^4 km, it seems reasonable to assume that the thickness of the diver is at least of the order of 10 km, and the assumption that $\beta < \omega$ appears justified.

If the short period motion is a natural oscillation, then the 90-day and 9-year components are likely to be forced oscillations resulting from cyclic changes in the vertical position of the diver. The mechanisms that might cause these forced oscillations were described earlier.

Estimates of the magnitudes of the vertical motions associated with the 90-day and 9-year oscillations can also be obtained from Eq. (21) and from the observed periods and longitudinal amplitudes. We assume that these oscillations are represented by the last two terms in (21). The decay constant in these terms (α) is a measure of the resistance to horizontal motion of the Red Spot. As noted earlier, the ratio of the amplitudes of the vertical and longitudinal oscillations is sensitive to the ratio of α to Ω. For the 9-year and 90-day oscillations, the known values of θ_i (the amplitude of longitudinal motion) and Ω_i are as follows:

for the 9-year oscillation

$$\theta_1 = 0.18 \text{ rad},$$
$$\Omega_1 = 0.0021 \text{ rad/day},$$

for the 90-day oscillation

$$\theta_2 = 0.018 \text{ rad},$$
$$\Omega_2 = 0.070 \text{ rad/day}.$$

θ_1 and θ_2 are equivalent to linear distances of 12,600 km and 1260 km, respectively. The vertical amplitudes, A_1 and A_2, of these motions are dependent on the value of α. We consider first certain broad limits imposed on these quantities by the nature of the model and by observational data.

An obvious limiting condition is $\alpha = 0$, which means that there is no resistance to the motion of a solid object through the fluid. This is essentially equivalent to the condition $\alpha \ll \Omega$, which means that drag forces, though finite, are small enough that the rate of decay of the horizontal motion is negligible compared to the frequency of oscillation. In this case we have, from Eq. (19):

$$|A_1| = \frac{R_0 \Omega_1 \theta_1}{2\Omega_J} \simeq 1 \text{ km}, \tag{30}$$

$$|A_2| = \frac{R_0 \Omega_2 \theta_2}{2\Omega_J} \simeq 3 \text{ km}. \tag{31}$$

These are minimum values. Equation (16) shows that an increase in α leads to increases in A_1 and A_2. The estimates from (30) and (31) emphasize again that minor vertical displacements can result in substantial displacements in longitude when damping effects are small.

An upper limit for A_1 and A_2 can be inferred from the fact that the latitude of the Red Spot does not undergo cyclic changes of the same frequency as the observed longitudinal oscillations. At the latitude of the Red Spot ($\sim 22°$) a vertical displacement of about 3500 km would result in an apparent change in latitude of $1°$, which would show up in a graph of latitude vs. time. Irregular fluctuations in latitude as large as several degrees do occur (see Fig. 3). However, the reported latitude is the average of the north and south edges and the latter do not correlate particularly well. Hence, the latitudinal changes may reflect more intense erosion of the diver by one or the other of the adjoining east–west currents. There are instances of fairly large, abrupt changes in latitude—one such is reported by Solberg (1968) as having taken place in January 1966—which appear to be due to interactions between the Red Spot and a smaller spot in one of the currents.

If the method used above to estimate β is applied to α, we find, for solid objects with dimensions of order of 100 km, a value of about 0.1 for α. This leads (Eq. 20) to the estimates $A_1 \simeq 40$ km and $A_2 \simeq 4$ km. These can be considered only very crude approximations. However, we note that the decay constant, α, would have to be small indeed to be negligible for the low frequencies encountered here. Hence, the latter estimate for the vertical motions appears to us to be more reasonable estimates for the A_i than are the values given by (30) and (31).

Turning now from these quantitative estimates we shall describe the qualitative behavior which is consistent with the analysis of the Red Spot motions treated as surface features attributable to a Cartesian diver.

Even though the higher frequency (7 and 12-day period) motions, which we have supposed to be due to a free oscillation associated with the different density gradients of the fluid and the solid, are rather irregular, they can be shown to be consistent with the present model. The lower frequency motions (90-day and 9-year period) are quite regular and will be discussed below.

In the analysis for the Red Spot motions we have employed a coordinate system in which longitude is measured positive in the direction of rotation of Jupiter. *It is important to keep in mind that System II longitude, in terms of which most of the observations are recorded, is measured positive in the opposite direction.* We shall suppose that the damping coefficient, α, is large relative to Ω_i so that the limiting form of the solution given by Eq. (18) is applicable to the motions with either the 90-day or the 9-year period. In this case the longitudinal displacement is 180° out of phase with the vertical displacement. Hence, keeping in mind the sign of System II longitude, we note that the Red Spot should show a maximum System II longitude at the point of highest elevation of the Cartesian diver.

There is, of course, no direct way to confirm this conclusion. We therefore turn to a physical discussion which is consistent with the physical processes proposed earlier. First, however, it is necessary to digress briefly to discuss the flow of fluid in the vicinity of a solid obstacle.

Many investigators have attempted to show experimentally how fluid in a rotating system flows around a solid object. Taylor (1923) was the first to exhibit the existence of a Taylor Column (as we now call it). In the papers cited earlier Hide and his co-workers have verified the existence of a Taylor Column.

In Fig. 8 we show trajectories of fluid flow past a conical object in a system rotating counterclockwise. This elegant illustration of the Taylor Column effect is taken from the work of Boyer and Vaziri (1970) who investigated the manner in which uniform flow upstream (to the left) of the cone is distorted as it passes over the cone.

Since the currents flowing past the Red Spot are westward near the southern edge of the Spot and eastward near the northern edge and since the normal com-

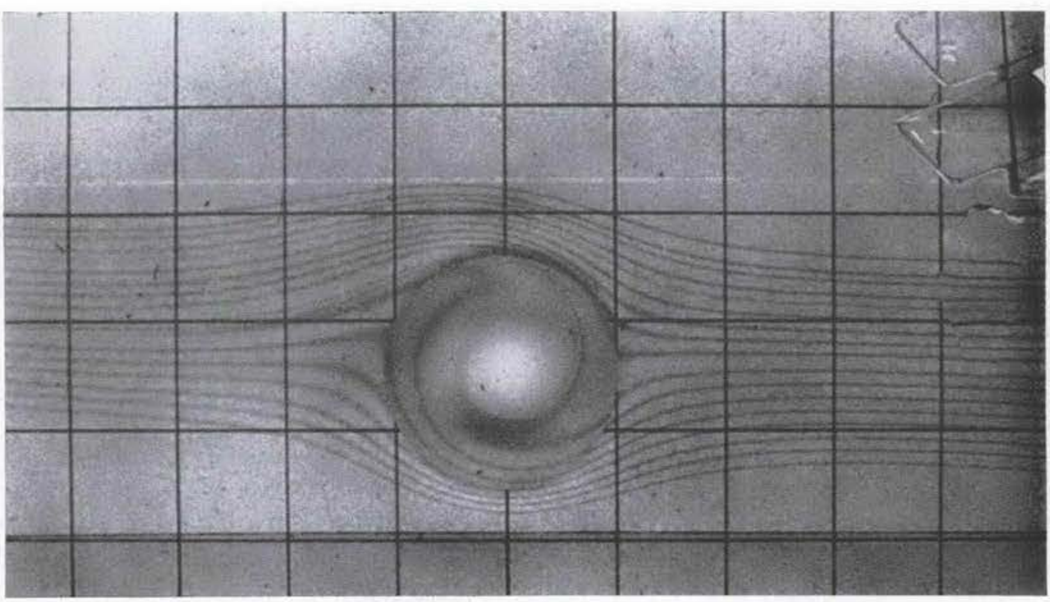

FIG. 8. Uniform flow, shown as straight line trajectories at the left, is distorted as it passes near and over a conical object. The resulting pattern of flow shows a clockwise circulation near the edge of the cone and a counterclockwise circulation near the center. (Rotation counterclockwise. The figure is Fig. 6 in the manuscript by Boyer and Vaziri (1970).)

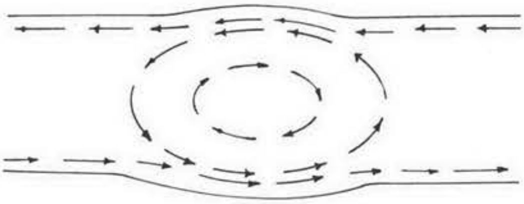

FIG. 9. Plausible pattern of flow above the Red Spot.

ponent of rotation is clockwise in the southern hemisphere of Jupiter, the resulting flow pattern over an assumed lens-shaped Cartesian diver will differ from that shown in Fig. 8. In Fig. 9 we have sketched trajectories under the foregoing conditions which are consistent with experiments and are qualitatively correct. Hence, the flow over a Cartesian diver would manifest itself as a counterclockwise vortex flow with the greatest intensity near the edge of the Red Spot, and a clockwise vortex near the center. Now imagine that the diver oscillates in the vertical direction. If the density gradient of the fluid is adiabatic, then it is easy to show that superimposed on the flow pattern of Fig. 9 would be an oscillatory circulation which, in a system rotating clockwise, would be counterclockwise when the diver is rising and clockwise when it is descending. This means that the vortex motion near the edge of the diver should result in particle motions which circle the region above the Red Spot faster during rising motion and slower during sinking motion. By our analysis we can transcribe "rising motion" to "increase in System II longitude."

This conclusion is borne out by the observations of vortex motions above the Red Spot as reported by Reese and Smith (1968). The spot which they called spot A was "trapped" above the Red Spot during a period of increase in System II longitude and it circled the Red Spot in 9 days. Spots B and C, as well as spots D and E, were all reported to have 12 day circulation periods and all were observed during a time of decrease in System II longitude. We conclude, therefore, that the basic period of circulation due to the zonal flow past the diver is approximately 10 days and the vertical oscillation of the diver shortens or lengthens the period to the values 7 and 12 days. Hence, these observations, though providing only indirect supporting evidence for our simple analysis, are at least consistent with the results.

In the discussion of Fig. 3 we noted that there is a strong correlation between changes in longitude and length for both the 9-year and 90-day periods and between longitude and width for the 9 year period. The length and width curves include

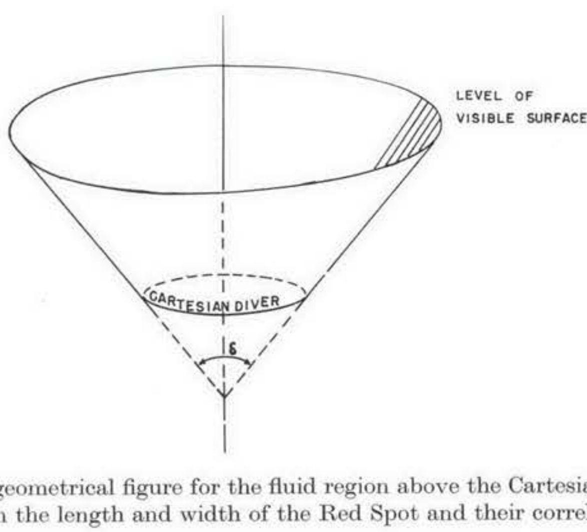

FIG. 10. A simple geometrical figure for the fluid region above the Cartesian diver, suggested by observed variations in the length and width of the Red Spot and their correlation with changes in longitude.

periodic components which are 180° out of phase with the corresponding components in the longitude curve. In the context of the Cartesian diver model these relations suggest a simple geometrical effect. The simplest figure which can account for the observed changes is an elliptical cone, as shown in Fig. 10. Let us assume that the major axis of a cross section of this cone is about twice the minor axis, and that the base of the cone is formed by its intersection with a fixed plane. Then motions of the cone along its axis will produce changes in the length and width of the base, such that the rate of change of length is twice that of the width. Now let us assume that a bounded fluid region, having this geometry, exists above the Cartesian diver in Jupiter's atmosphere and that its intersection with the upper cloud layers forms the visible Red Spot. Consider, then, the effect of a vertical displacement of the Cartesian diver on the longitude, length and width of the visible spot. As the diver rises its horizontal velocity decreases, due to conservation of angular momentum, resulting in an increase in its System II longitude. Simultaneously, both length and width should decrease. When the diver descends, the reverse should occur. Indeed, these are precisely the effects shown in Fig. 3.

An estimate of the angle of the cone (δ in Fig. 10) can be made from the observed changes in length and width and from estimates of the corresponding vertical displacements. The relation is

$$\tan\frac{\delta}{2}=\frac{\Delta L}{2A},$$

where ΔL is the net change in length over one half cycle and A is the amplitude of the vertical oscillation. For both the 90-day and 9-year oscillations we find $175^\circ > \delta > 180^\circ$ for values of A of several hundred kilometers or less.

A second interpretation of the observed changes in longitude and size of the Red Spot follows from a consideration of certain physical and thermodynamic properties of the system. We have speculated that changes in the size of the spot over the 9-year period may be caused by erosion of the solid material as the diver rises, and resolidification as it descends. Again, the size of the spot would be 180° out of phase with the longitude, as shown in Fig. 3. We proceed with our speculative interpretation by noting that fluid flow over a floating object such as a Cartesian diver would show up as a Taylor Column in a rotating homogeneous fluid. If the fluid has variable density but the density gradient is adiabatic, as we suppose to be the case here where there is an excess of outgoing to incoming heat flux, a Taylor Column would still form. For such a shallow system (shallow in the sense that the depth of the fluid is comparable to or smaller than the horizontal scale of the object) the Taylor Column is essentially vertical. Since the diver serves to shield the overlying atmosphere from upward heat flux and its associated cloud structure, an observer should be able to see deeper into the atmosphere in this region. This, as we proposed earlier, accounts for the darker color over the Red Spot. As the diver oscillates vertically, energy will propagate from the edges of the object along the characteristics whose direction is determined by the properties of the fluid and by the frequencies of oscillation and of basic rotation. For the present case we have a very low oscillation frequency ($\Omega_i \ll \Omega_J$) and the propagation direction in a rotating homogeneous fluid is essentially vertical, i.e., we have a quasi-static Taylor Column. Therefore, the size of the Taylor Column simply reflects the size of the Cartesian diver.

In addition to the foregoing, however, we should also observe that the upper Jovian atmosphere is (very probably) stably stratified. When the characteristics pass from the adiabatic to the stably stratified region, they are turned toward the horizontal. The degree of this bending is determined by the stratification. It appears to us very probable that the upper atmosphere of Jupiter can be considered to be very stable in the sense that the characteristic surfaces along which energy is propagated are practically horizontal. The approximate horizontal cross section of the Red Spot is elliptical with the major (east–west) axis about twice the minor axis. Hence, the characteristic surfaces will be two elliptical cones, to a first crude approximation, one cone having its apex in the adiabatic region and expanding upward and the other cone with its apex in the stably stratified region or outside the Jovian atmosphere and expanding downward. The configuration is shown in Fig. 11. The two cones intersect and have equal cross sections at the level where the gradient changes from adiabatic to subadiabatic. Energy is propagated along the parts of the surfaces in the stably stratified region.

As the diver sinks and expands, the Taylor Column above it will expand and so will the observed elliptical cross section of the cone which opens upward. Hence, the observed Red Spot would increase in size in such a manner that the rate of increase of the length is twice the rate of increase of the width. Corresponding contraction can be expected when the diver rises. A two-to-one ratio of rates of increase of length to width is approximately what is shown by the curves for width and length of the Red Spot in Fig. 3.

As we pointed out earlier, the massive disintegration of the diver along its edges when the object is near its high point will be accompanied by a relatively abrupt release of buoyant fluid and a consequent upward flux of this fluid near the periphery of the diver. When this material reaches the stably stratified level it will tend to penetrate into it. The resulting disturbance will be propagated upward not only along both of the conical characteristic surfaces, but the region enclosed by these surfaces should also show a cloudiness because of the penetration of buoyant fluid. Hence, the Red Spot should fade considerably during this period. Solberg (1968) reported that the Red Spot was indeed very much less visible and considerably smaller during the 1966–67 apparition. At this time it was in a position of maximum System II longitude. In the next (1967–68) apparition the Red Spot increased in size and clarity and its System II longitude decreased. Hence, the observations fit the present picture.

A final point which is worth noting is that the characteristic surface which has the form of an elliptic cone expanding downward might have its apex above the Jovian atmosphere. Hence, disturbances propagated along this surface should be visible in an elliptical ring contained within the observed Red Spot. In a recent publication Reese (1969*a*) shows such an inner ring as part of the sketches of the Red Spot. If our present speculation is correct, the inner ring should contract and expand in the same manner as the Red Spot. If the alternative, simple geometric, picture offered earlier is correct

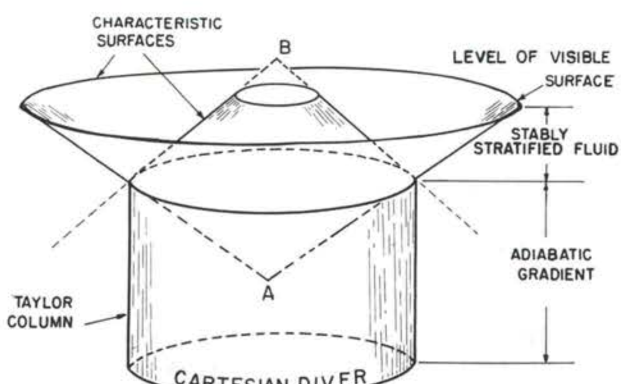

FIG. 11. A Taylor Column forms as a vertical column over the Cartesian diver in the fluid with adiabatic gradient. When the column intersects a region of stably stratified fluid, energy is propagated along characteristic surfaces, one expanding upward and outward and the other contracting upward and inward from the intersection level.

and if an analogous downward expanding cone were added to explain the inner ring, the latter should expand when the Red Spot contracts and vice versa.

SUMMARY AND CONCLUSIONS

Experimental phase diagrams for binary mixtures of light gases, extrapolated to very high pressures, suggest that solid–fluid phase separations exist over broad ranges of pressure and temperature in such mixtures. There is further evidence that the solidification of hydrogen–helium mixtures is accompanied by an inversion in the relative densities of coexisting solid and fluid phases on crossing a fixed boundary in pressure–temperature space. When applied to the atmosphere of Jupiter, these phase diagrams lead to a model for the deep atmosphere in which masses of hydrogen-rich molecular solid are neutrally buoyant at a certain level within the surrounding fluid mixture of hydrogen and helium. We have postulated that the Red Spot is the visible manifestation of a large, though not necessarily continuous, mass of this floating solid, which would exhibit many of the characteristics of a Cartesian diver.

This physical picture has been used to develop a simple mathematical model, for the motions of the Red Spot, based on the motions of a thermodynamically stabilized Cartesian diver in a rotating system. Qualitatively, the motions predicted by this model are remarkably similar to the observed longitudinal motions of the Red Spot, and quantitative estimates of relevant physical properties of the system lead to reasonable values for the underlying vertical motions. We have suggested several possible mechanisms for the observed oscillatory motions of different periods.

Although the Cartesian diver hypothesis is based to a large extent on speculations concerning the behavior of matter at high pressures and the thermodynamic and physical properties of Jupiter's deep atmosphere, it nevertheless provides a reasonable and consistent explanation for many of the observed characteristics of the Red Spot. It is clear that if the physical picture of the Cartesian diver is correct, there is good reason to believe that floating masses of hydrogen-rich solid can exist in all latitudes on Jupiter, and it is reasonable to ask whether certain of the observed transient spots are not manifestations of the same phenomenon on a smaller scale. In this connection it is interesting to note that Peek (1958) reported several transient spots whose motions in longitude include an oscillatory component, and similar observations have been reported recently by the group at New Mexico State University (Reese and Solberg, 1969; Reese, 1970). Reese and Solberg (1969) have suggested that oscillatory components in the motions of transient spots may have been missed in the past because of insufficiently accurate data. It follows that small oscillatory components may be obscured by scatter in the data.

This discussion of the Cartesian diver hypothesis suggests several areas for further studies aimed at clarifying both the physical principles on which the hypothesis is based and the nature of the motions of the Red Spot and the transient spots. Further laboratory studies of the phase behavior of light gas mixtures are clearly indicated, and we are now preparing to carry out experiments on hydrogen–helium mixtures at pressures up to 10,000 atm. These experiments should provide quantitative data on the solidification of hydrogen–helium mixtures at moderate pressures, and on the conditions under which an inversion occurs in the relative densities of coexisting solid and fluid phases. Laboratory and theoretical studies of fluid flow in the vicinity of oscillating bodies in rotating systems should also help to clarify relevant dynamical principles. On the observational side, it will be of interest to see whether the several oscillatory components in the Red Spot motion continue to manifest themselves, and whether the motions of other transient spots can be related to the Cartesian diver model. We are presently studying the possibility of obtaining further information on the periodic components of the

Red Spot motion through a numerical analysis of the original observational data supplied to us by E. J. Reese of the New Mexico State University Observatory. This problem is especially difficult because of the irregular spacing of the data.

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