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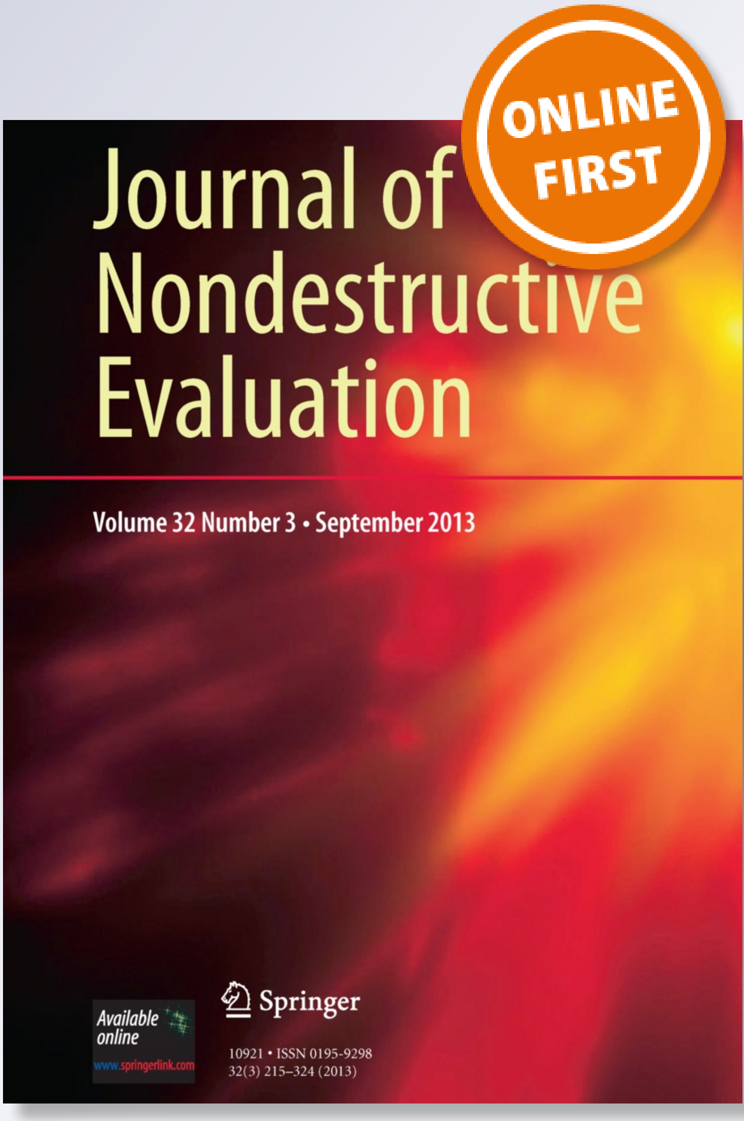


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# Dethermalization Theory: Thermal/Dielectric Analogy for Heat Flow Through Porosity in Composites

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**Abstract** The flow of heat in solids has long been known to possess an electric current analogy applicable to both steady state and transient flows. In the present work we assume a vector analogy between Fourier's law and the classical electric displacement to develop a method of handling distributed porosity in composite materials subject to heat flow in a way analogous to dealing with distributed dielectric regions in solids subject to an external electric field. The effect of the geometry of "depolarization" regions in an electric displacement field and "demagnetization" regions in a magnetizing field can be carried over to the effect of "dethermalization" regions in a heat-flux field. The analogy provides a simple analytic way of determining the effects of porosity shape on thermal conductivity which can be significant and can violate the usual law of mixtures approach. For uniformly distributed porosity of known aspect ratio in a given region, the volume-fraction porosity of the region can then be evaluated from a simple measurement of the thermal diffusivity. This approach was originally successfully tested over a limited range of variables when the model was developed and has recently been validated to good accuracy over a large range of porosity aspect ratios.

**Keywords** Thermal charge · Thermal dipole · Thermal-dielectric analogy · Thermal diffusivity · Porosity · Composites

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## 1 Introduction

Porosity evaluation plays an important role in the determination of the integrity of composite materials. For example, porosity content is known to reduce strength [1]. On the positive side, controlled porosity can alter the thermo-physical properties of materials such as insulation. In either case it is useful to have a rapid means of measuring porosity content other than metallographic preparation. Flash thermography provides a simple technique to evaluate porosity in structural composites ranging from carbon fiber-reinforced plastics (CFRP) to ceramics. A variety of such methods have long been available, but all the quantitative approaches have one thing in common, measurement of the thermal diffusivity. Thermal diffusivity is defined as

$$\alpha = \frac{K}{\rho c},$$

where $K$, $\rho$ and $c$ are, respectively the thermal conductivity, density and specific heat. The goal of the present work is to understand how each of these thermal parameters is influenced by the presence of porosity and how to deal with it.

## 2 Dethermalization Theory

### 2.1 Law of Mixtures

In general we are dealing with the thermal evaluation of porosity in a thermally anisotropic material. However, we will limit ourselves with through-thickness heat flow—that is, flow perpendicular to the fiber planes in, for example, graphite-epoxy composites. It is reasonable to expect that the effect of porosity is simply to reduce the thermal conductivity of the material in some average way as well as to

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reduce the density. We also know that simple scalar properties such as density can be treated with a Law-of Mixtures (LOM) analysis which states that the average density is the volume-fraction-weighted sum of the densities of the constituents. Using the subscript $M$ for matrix (for example the graphite-epoxy averaged material property) and P for porosity (air) where $V$ is the volume fraction, we have, for a 2-phase mixture:

$$\rho = V_M \rho_M + V_P \rho_P; \qquad V_M + V_P = 1.$$

If we take the specific heat to be essentially invariant in the averaged medium, we can write the density specific heat product (volumetric heat capacity) as a LOM:

$$(\rho c)^* = (1 - V_P)(\rho c)_M + V_P (\rho c)_P. \tag{1}$$

Terms of order $V_P \rho_P$ can be ignored for air or vacuum.

### 2.2 Electromagnetic Heat Flow Analogies

Thermal conductivity is considerably more complex than density since it has a tensorial nature depending on pore shape and heat flow direction. We have shown in an earlier study [6] of porous CFRPs, completed in 1994, that density/porosity fits a LOM model while thermal conductivity does not. In that work we examined porosity standards made for us by Celsius Materialteknik AB of Sweden and Sikorsky of Stratford, CT. These samples included quasi-isotropic, unidirectional and weave layups. Porosity was analyzed quantitatively ultrasonically and by sectioning. The intent of the study was to use flash thermography for the first time to quantitatively evaluate porosity. This work incorporated the first known use of the focal plane array IR camera for NDE. At that time the camera, made by Amber Eng., was liquid nitrogen cooled with a $128 \times 128$ pixel InSb array.

It was clear that thermal conductivity is a complicated property of the CFRP matrix/porosity mixture and depended on pore shape. However, similar work has been done with dielectric mixtures and electrical conductivity. It has long been known that there is a strong analogy between heat flow and electric current flow based on the conduction equation, compared to Fourier's law of heat flow:

$$\boldsymbol{J} = \sigma \boldsymbol{E} = -\sigma \nabla \varphi$$

$$\boldsymbol{f} = k \boldsymbol{F} = -k \nabla T$$

Bold letters represent vectors. $\boldsymbol{J}$ is the electric current, $\boldsymbol{E}$ the electric field, $\boldsymbol{f}$ is the heat flux and $k$ is the thermal conductivity analogous to the electrical conductivity, $\sigma$. Temperature and potential are therefore analogs. This analogy is obvious, but the electrical side holds only for conductors. For electric insulators (dielectrics), electric displacement, $\boldsymbol{D}$, replaces current and we can build a second analogy from this. The normal component of electric displacement, $\boldsymbol{D}$, is continuous across a boundary between two dissimilar materials as is the heat flux vector, $\boldsymbol{f}$. While the electric displacement is induced by an electric field, (a potential gradient), the heat flux is induced by a temperature gradient. Temperature and potential are analogs as before.

The importance of this analogy derives from the failure of thermal conductivity to follow a LOM. A dielectric analogy might provide an established method for handling porosity in matrix materials. Dielectric and magnetic permeability studies have been around a long time and, in particular, complex electrical analyses of mixtures consisting of a matrix with a dilute embedded material are referenced in [2]. These studies indicate that the shape of the embedding plays an important role in determining the average electric or magnetic field in the material. An applied electric or magnetic field in the material will polarize or magnetize the embedding. The shape of the embedding will effect a depolarization or demagnetization due to the presence of reverse-induced dipoles, a classic result first noted by Bozarth and Chapin [3] and discussed, for example, in Kittel's Introduction to Solid State Physics [4]. The relationship to thermal work was first suggested by D.K. Hale [2] and refers to the demagnetization analyses of Stoner [5]. We were able to successfully translate these electromagnetic approaches for dielectric inclusions to thermal approaches. These results were first validated by the data fits we obtained [6] for a variety of volume fraction porosities of a given shape—fits that could only be found given the dielectric analogy. Details of this first validation will be described in Sect. 3 along with the most recent tests of Mayr et al. [8]. We present a review below for reference in understanding the thermal/dielectric analogy.

### 2.3 Choice of Units

Although SI units are in common use, for electromagnetic theory different units are often chosen depending on convenience for a particular application. For the electromagnetic work below, we choose electrostatic (esu) units to avoid the constant presence of the vacuum permittivity and permeability ($\varepsilon_0$ and $\mu_0$ respectively). This requires that we translate the analogy to heat using cgs units. One can, of course easily convert any results to SI units. For convenience we define all thermal units below:

Thermal conductivity $k$ (cal/s cm °C); density $\rho$ (g/cm$^3$); specific heat $c$ (cal/g °C).

In these units, the thermal conductivity of the best metallic conductor, silver, is approximately 1.0 cal/s cm °C, thus allowing for straightforward approximation assumptions.

### 2.4 The Dielectric Analogy

The embedded dielectric is taken to be oblate spheroids of revolution since these closely mimic the geometry of "squashed" porosity in ply layups. Actual porosity geometry will vary. The electric displacement, $\boldsymbol{D}$, in a dielectric medium arising from an applied electric field, $\boldsymbol{E}$, in the dielectric, is given by:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = -\varepsilon \nabla \varphi, \tag{2}$$

where $\varepsilon$ is the dielectric constant of the medium. This is to be compared, as before, to Fourier's law,

$$\boldsymbol{f} = k\boldsymbol{F} = -k\nabla T, \tag{3}$$

We shall refer to $\boldsymbol{F} = -\nabla T$ as the thermal field. The electric displacement in the medium is also the sum of the electric field, $\boldsymbol{E}$, and polarization, $\boldsymbol{P}$, induced by the field acting on bound surface charges:

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi \boldsymbol{P}, \quad \boldsymbol{P} = \chi \boldsymbol{E}, \tag{4}$$

where $\chi$ is the electric susceptibility. Analogously, we may define a thermal displacement and polarization field as:

$$\boldsymbol{f} = \boldsymbol{F} + 4\pi \boldsymbol{P}_T, \quad \boldsymbol{P}_T = \chi_T \boldsymbol{F}. \tag{5}$$

From (2) and (4), the dielectric constant is related to the electric susceptibility:

$$\varepsilon = 1 + 4\pi\chi \tag{6}$$

Thus we can define a "thermal susceptibility" related to the conductivity from (3) and (5):

$$k = 1 + 4\pi\chi_T. \tag{7}$$

We note the thermal susceptibility is generally negative, for example, in cgs units, the most thermally conductive metal is silver with $k = 1.0$ cal/s cm °C. Thus $k \le 1.0$.

In practice, the geometry of the electromagnetic medium, which will affect the distribution of induced dipoles on the surfaces, will introduce a "depolarization" field, $-4\pi\eta_i \boldsymbol{P}$ which is added to the applied field (field without dielectric), $\boldsymbol{E}_0$, thus defining the internal field, $\boldsymbol{E}$, in the dielectric. $\eta_i$ is the "depolarization" factor.

$$\boldsymbol{E} = \boldsymbol{E}_0 - 4\pi\eta_i \boldsymbol{P}. \tag{8}$$

The thermal analogue, due to the negative $\chi_T$, must have the opposite sign.

$$\boldsymbol{F} = \boldsymbol{F}_0 + 4\pi\eta_i \boldsymbol{P}_T. \tag{9}$$

Thus the thermal polarization will act to "dethermalize" the region by reducing the heat flux. We may even refer to "thermal dipoles" as defined by hot (+) and cold (−) "charges" bounding a region, the thermal field defined pointing from hot to cold. This can be hypothesized for two reasons: (1) Point sources and sinks are describable by means of Green's functions in the heat equation as discussed, for example, in Carslaw and Jaeger [10]. (2) The present theory has been validated both by Ringermacher et al. [6] and Mayr et al. [7, 8], thus supporting the concept of thermal "charge distributions". More comments will follow later.

The depolarization factor is strictly a geometric quantity related to the shape of the embedding and defined from a weighted sum over spatial orientations of the ellipsoids:

$$\eta = \sum_{i=1}^{3} \eta_i = 1. \tag{10}$$

This is then directly transferable as a "dethermalization factor" in heat flow. Thus, if the direction perpendicular to the plane of rotation of the ellipsoid is defined as longitudinal and in-plane as transverse then we have for the case of cylindrical symmetry with two equivalent in-plane orientations:

$$\eta_L + 2\eta_T = 1. \tag{11}$$

For the case of spherical symmetry with $\eta_L = \eta_T$, we have $\eta_L = \eta_T = 1/3$. The general formula for $\eta_L$, taken directly from electromagnetic theory, depends on the "aspect ratio", $m$, (long/short axis), of the ellipsoid, where the short axis, along which the heat flows, is "longitudinal":

$$\eta_L = \frac{m^2}{(m^2-1)} - \left[\frac{m^2}{(m^2-1)^{3/2}}\right] \sin^{-1}\left[\frac{(m^2-1)^{1/2}}{m}\right]. \tag{12}$$

The $\eta_i$ of Eq. (8) is taken along the direction of the applied field. Thus a thin disk with plane oriented perpendicular to the field will induce the maximum longitudinal depolarization with $\eta_T = 0$ and $\eta_L = 1$ while a long needle oriented along the field will have the least depolarization effect with $\eta_L = 0$.

The depolarization effect is equivalent to a change in the effective dielectric constant of the medium as is evident from Eqs. (2), (4) and (6). It has been shown [5] that the effective dielectric constant of a medium consisting of a mixture of a matrix with dielectric constant $\varepsilon_2$ and ellipsoids of revolution of dielectric constant $\varepsilon_1$ distributed and oriented randomly (Fig. 1b) is given by

$$\varepsilon^* = \varepsilon_2 + \frac{V_1(\varepsilon_1 - \varepsilon_2)}{3} \sum_{i=1}^{3} \left[\frac{\varepsilon_m}{\varepsilon_m + \eta_i(\varepsilon_1 - \varepsilon_m)}\right], \tag{13}$$

where $\varepsilon_m$ is the effective mean value of the dielectric constant of the medium around each particle and $\eta_i$ are the depolarization factors along the three ellipsoidal axes.

If we choose to evaluate ellipsoids that are randomly distributed but oriented with their axes of rotational symmetry normal to the plane (as is the actual case for most porosity—Fig. 1c) we simply drop the factor of three spatial average and use the depolarization factor, $\eta_L$, along the applied field

$$\varepsilon^* = \varepsilon_2 + V_1(\varepsilon_1 - \varepsilon_2)\left[\frac{\varepsilon_m}{\varepsilon_m + \eta_L(\varepsilon_1 - \varepsilon_m)}\right]. \tag{14}$$

Equation (14) is quite sensible, for when $\eta_L = 0$, for the case of long needles aligned along the field, this reduces to the standard LOM equation, $\varepsilon^* = V_1\varepsilon_1 + V_2\varepsilon_2$.

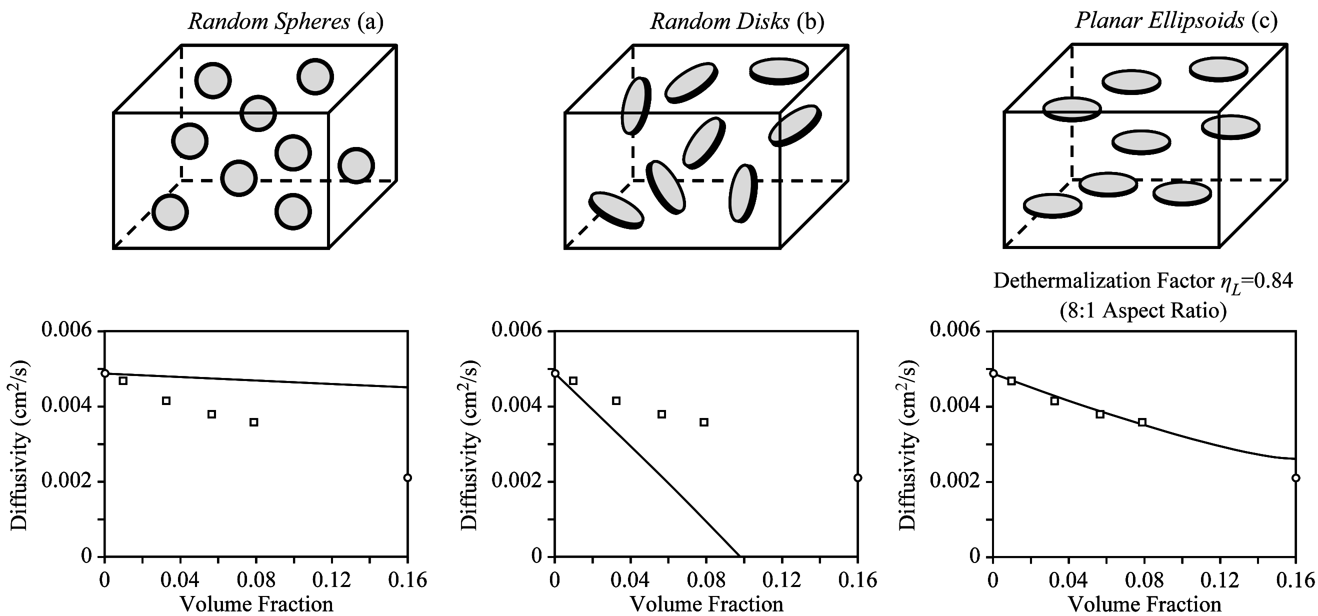


**Fig. 1** Effects of pore shape on fit of CFRP data from Ref. [6] to the dethermalization model of porosity comparing results of using Eq. (15) with the added $1/3$ factor in (**a**) and (**b**) and with exactly Eq. (15) in (**c**)

The thermal analogy simply requires replacement of dielectric constant, $\varepsilon$, with the thermal conductivity, $k$, as described in Eqs. (6) and (7) leaving us with the desired result for the present work:

$$k^* = k_M + V_P(k_P - k_M)\left[\frac{k_m}{k_m + \eta_L(k_P - k_m)}\right]. \tag{15}$$

Here, $\eta_L$, defined by (12), is the "dethermalization factor". $k_M$ is the matrix thermal conductivity and $k_m$ is now the mean thermal conductivity of the medium around each particle. We use the lowest order approximation that $k_m \cong k_M$ which should be taken as the weighted average of the thermal conductivity for a generally orthotropic medium. This will depend on ply layup.

We can finally write the effective thermal diffusivity of a porous medium as:

$$\alpha = \frac{k^*}{(\rho c)^*} \tag{16}$$

where the volumetric heat capacity, from Eq. (1), is given by:

$$(\rho c)^* = (1 - V_P)(\rho c)_M + V_P(\rho c)_P. \tag{17}$$

The above thermal equations form the basis of what we call the "Dethermalization Theory" of porosity [6]. From these, we can now estimate what volume fraction porosity might cause a measured thermal diffusivity, $\alpha$, given a known porosity aspect ratio, $m$.

## 3 Validation

The work described above was originally developed and tested, as described in Ref. [6], for studies of porosity in CFRP. In that work, the samples mentioned in Sect. 2 were prepared using an autoclave process resulting in "pancake" ellipsoidal-shaped interlaminar porosity. Porosity volume fractions and shape aspect ratio were measured from sectioning and ultrasonics. It was found that the average porosity had an aspect ratio, $m = 7 \pm 2$. The samples used in this early work had volume fraction porosities varying from 0.001 (control) to about 0.06 with one specimen at 0.16. The dethermalization model was fitted to test different distributions of porosity by plotting the measured through-thickness thermal diffusivity against the known volume fraction porosity in the various samples. Density and specific heat used in the calculation of diffusivity from Eq. (16) assumed a LOM. Figures 1a and 1b show thermal diffusivity vs. volume fraction porosity fits for random spheres and randomly oriented ellipsoids, respectively using Eq. (15) for the thermal conductivity but with the extra $1/3$ factor as in the analogous Eq. (13). The curves were normalized to the fully dense specimens. Neither one even remotely fits the porosity range. Figure 1c shows the fit for random planar-oriented ellipsoids using exactly Eq. (15) for the conductivity. This best fit is for an ellipse aspect ratio of $m = 8$ and dethermalization factor of $\eta_L = 0.83$, from Eq. (12), and is shown in more detail in Fig. 2. This compares well with the measured aspect ratio $m = 7 \pm 2$, thus confirming the assumptions of the model.

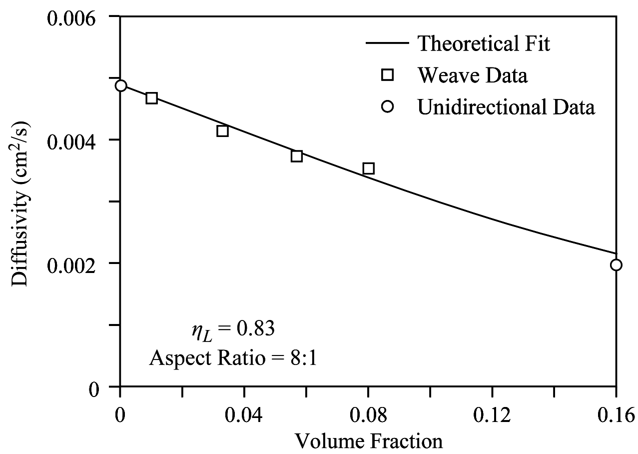


**Fig. 2** Data fit, using Eqs. (15) and (16) from Ref. [6] for theoretical porosity aspect ratio of 8:1 resulting in a dethermalization factor of 0.83. Measured aspect ratio was 7:1

Equation (15) has also recently been tested and validated by Mayr et al. [7, 8]. They utilized pulsed thermography to measure thermal diffusivity in samples of CFRP with embedded porosity. By changing the volume % porosity from 0.3 % to 10.5 %, they were able to adjust pore aspect ratio over a range of $m = 1$ to $m = 5$, thus testing the aspect ratio variation which our original work lacked. They measured the pore aspect ratio from CT imaging. In practice, since heat flows around pores, a weighted average of the in-plane and perpendicular matrix conductivities, depending on ply layup, should be used for $k_m$, the "mean" conductivity, as was demonstrated by Mayr. They obtained a very good fit of the diffusivity as a function of volume % porosity, a steeply falling curve toward large aspect ratio. The least-squares error to the above model was approximately $\pm 3$ % over the measured range along the curve. An alternative analysis by Kerrisk [9], based on an assumption of spherical voids only and corrected LOM for thermal conductivity, was shown by Mayr to produce a relatively flat curve, in agreement with the present theory for the spherical limit $m = 1$, but approaching errors as high as 30 % for larger aspect ratios, consistent with a LOM failure.

## 4 Conclusions

We have developed a methodology for quantitative analysis of measured thermal diffusivity in porous materials that takes into account the shape of the porosity affecting heat flow. The methodology is based on an exact analogy between electric displacement in dielectric materials and heat flux in thermally conductive materials through Fourier's law. As a result, since extensive analyses of electrostatic effects in dielectrics exist in the literature, these can be directly translated to their thermal equivalents. In the present work we have shown that thermal conductivity is the analogue of the dielectric constant and were able to apply the well-known theories of depolarization effects arising from dielectric embedding shape, to "dethermalization" effects arising from porosity shape for heat flow. A simple formula relates volume fraction porosity to a measured thermal diffusivity, given a known porosity aspect ratio.

Since our work was based on precise electromagnetic analogies to polarization effects utilizing electric charge and electric dipoles, it would suggest that these concepts can be extended to heat flow in insulative materials in the form of thermal charge and thermal dipole distributions. This is reinforced by the successful testing of the thermally-generalized results described above. Extensions of this work in new directions are made possible by utilizing these proven concepts.

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